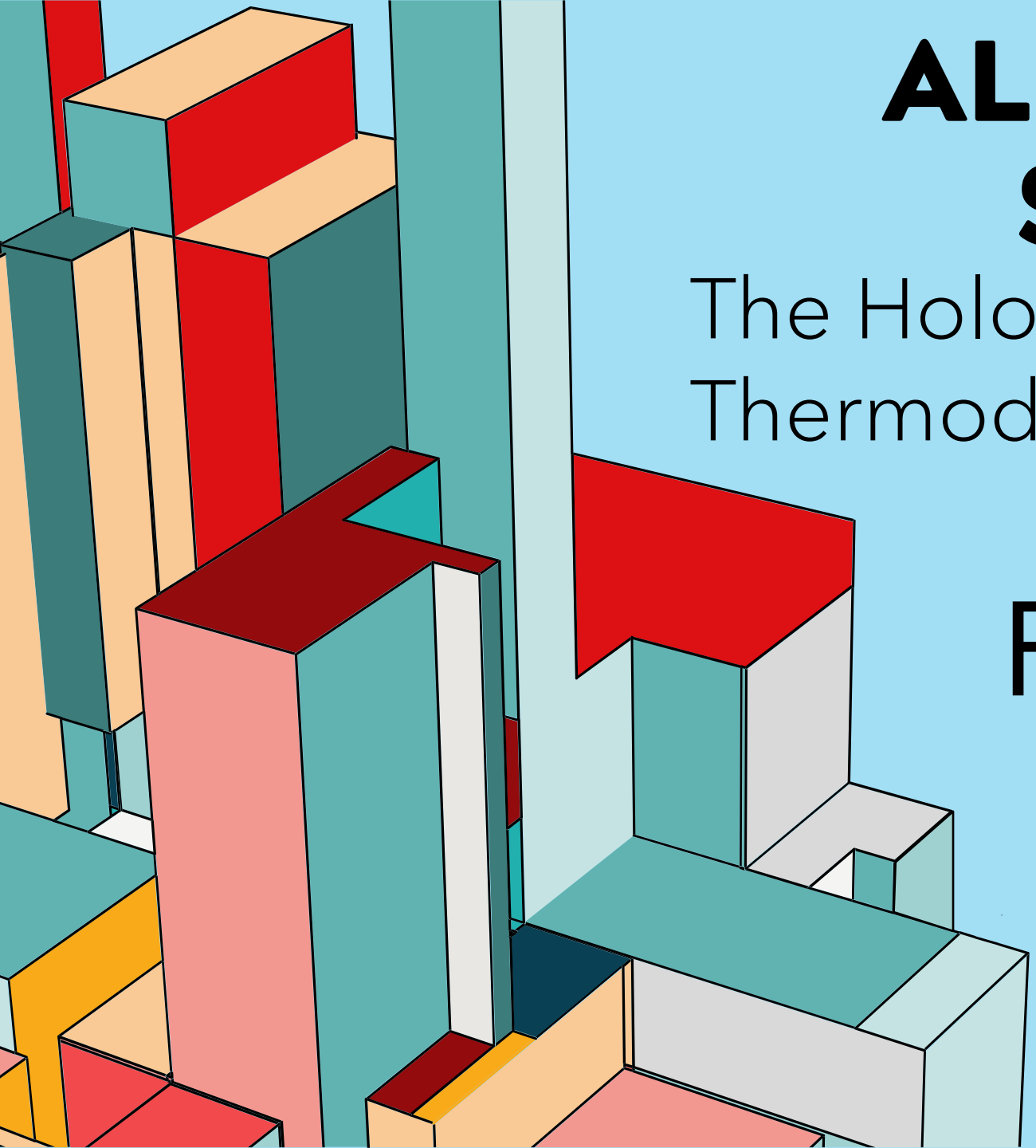


ALL SURFACE, NO SUBSTANCE?

The Holographic Thermodynamics of Black Holes

FYSICA2026

Silvester Gerard Adriaan Borsboom

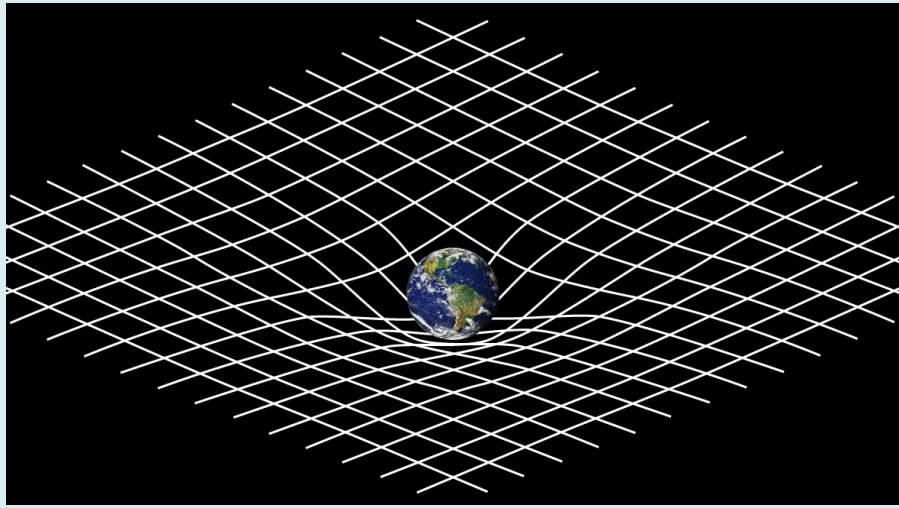
Based on: ArXiv 2602.05130

THE RADBOUD CENTER FOR NATURAL PHILOSOPHY (RCNP)

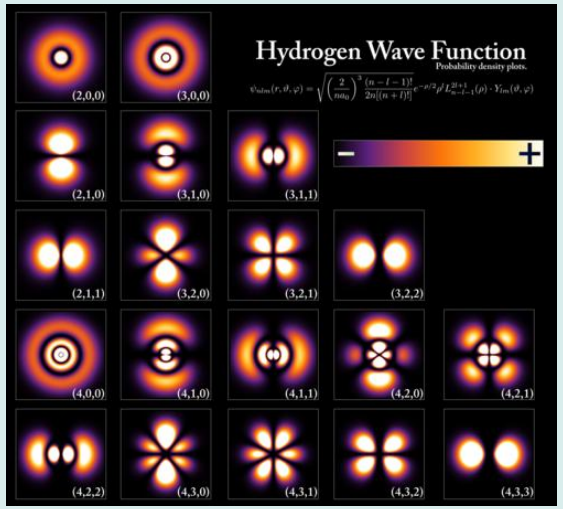
rcnp.science.ru.nl

THE TWO PILLARS OF MODERN PHYSICS

General relativity:

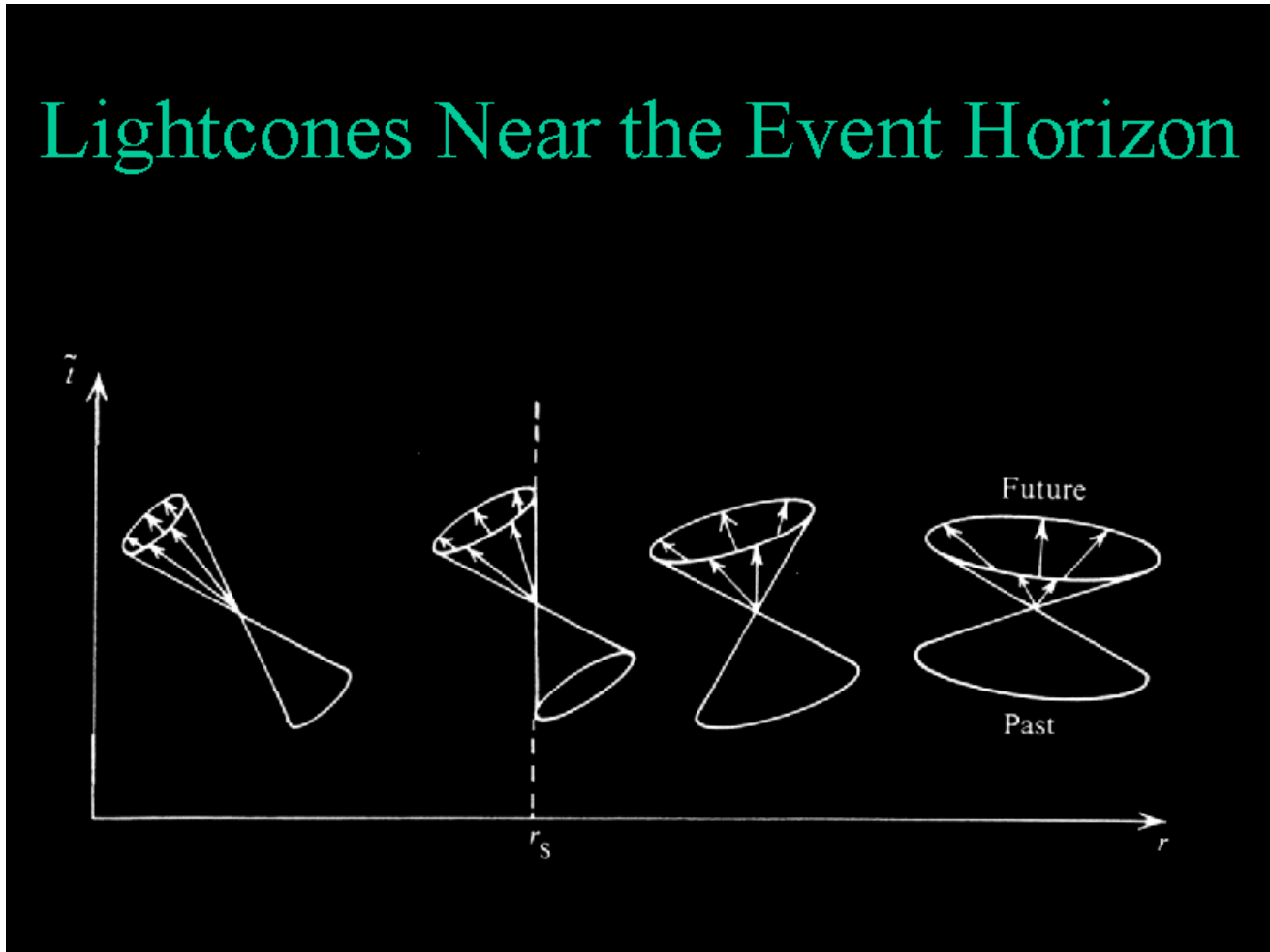


Quantum theory:



BLACK HOLES

- Event horizon
- Singularity
- Schwarzschild
- Rotating/charged
- No hair theorem

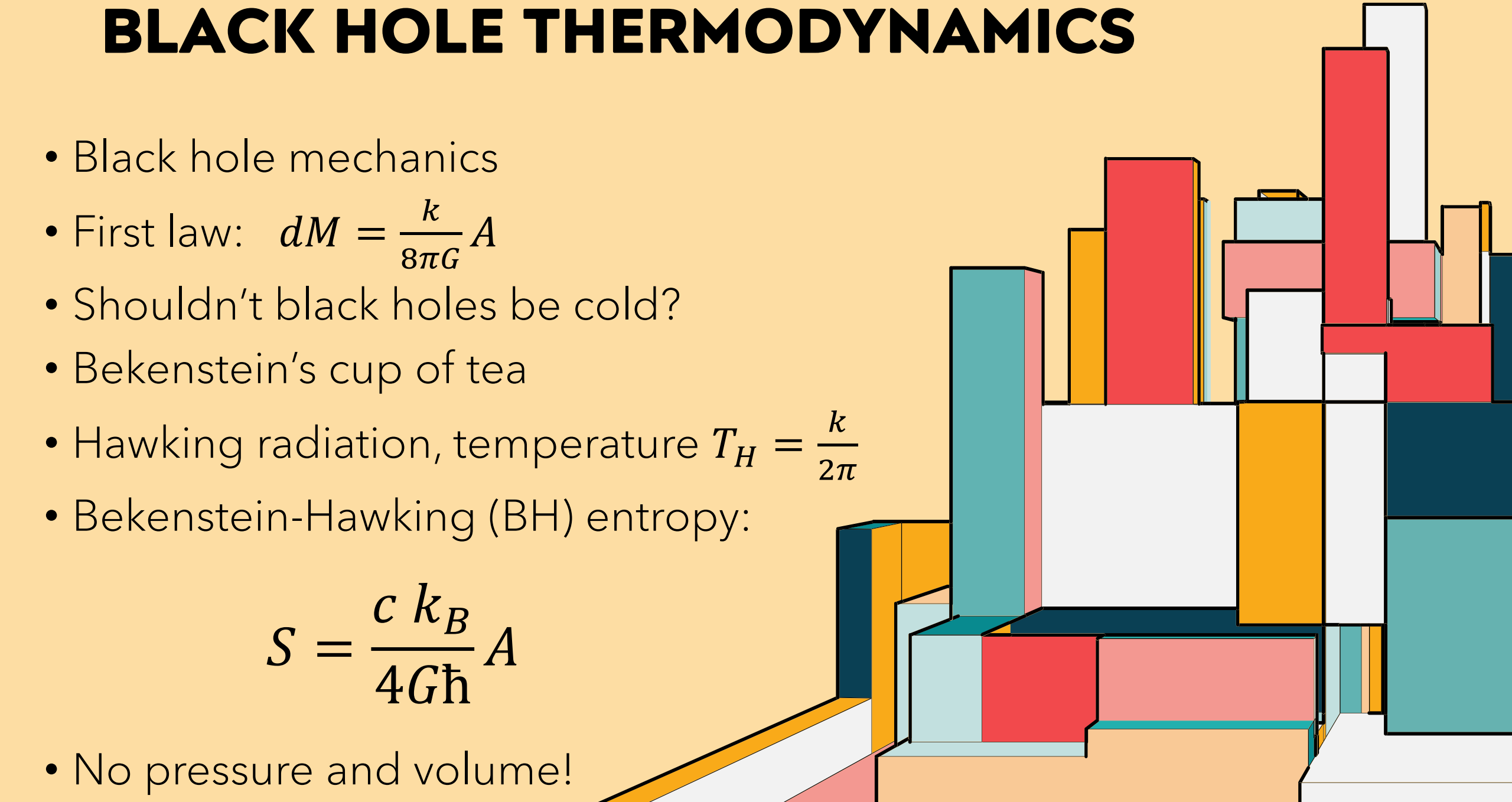


BLACK HOLE THERMODYNAMICS

- Black hole mechanics
- First law: $dM = \frac{k}{8\pi G} A$
- Shouldn't black holes be cold?
- Bekenstein's cup of tea
- Hawking radiation, temperature $T_H = \frac{k}{2\pi}$
- Bekenstein-Hawking (BH) entropy:

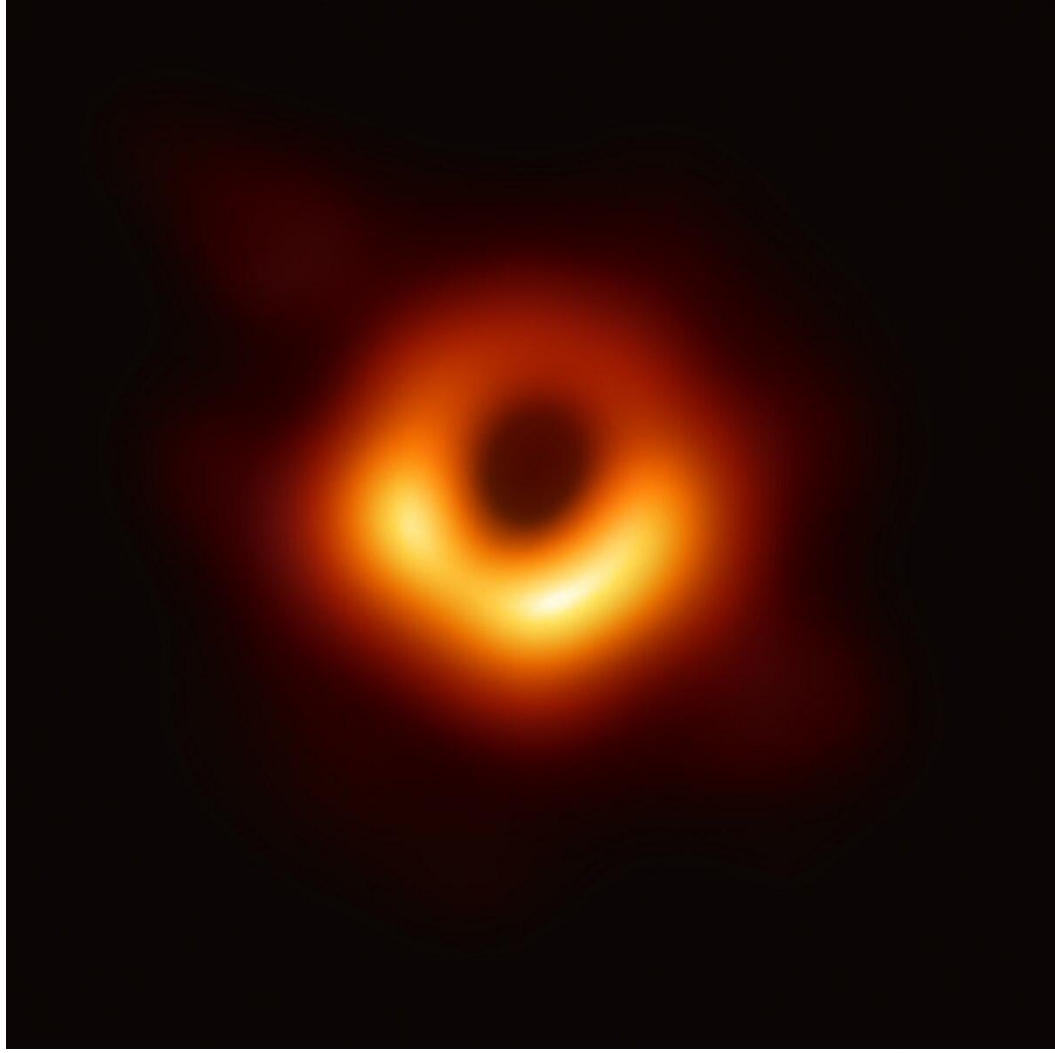
$$S = \frac{c\, k_B}{4G\hbar} A$$

- No pressure and volume!



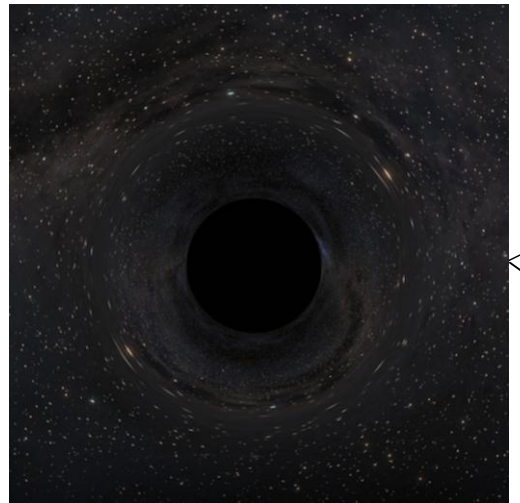
PREVIOUS PROPOSALS

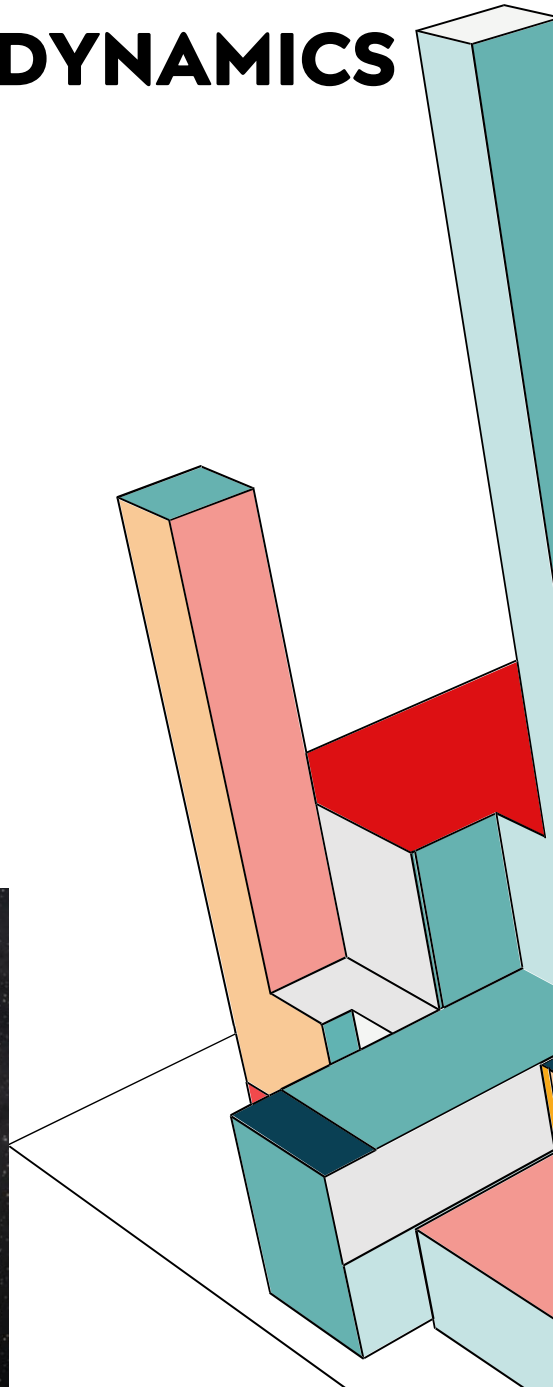
- Padmanabhan: use naïve Euclidean volume
- Thermodynamic quantities not independent!
- Black hole chemistry: use cosmological constant
- Degeneracy for AdS-Schwarzschild
- Varying Λ meaning changing the theory!



SURFACE CHARACTER OF GRAVITATIONAL THERMODYNAMICS

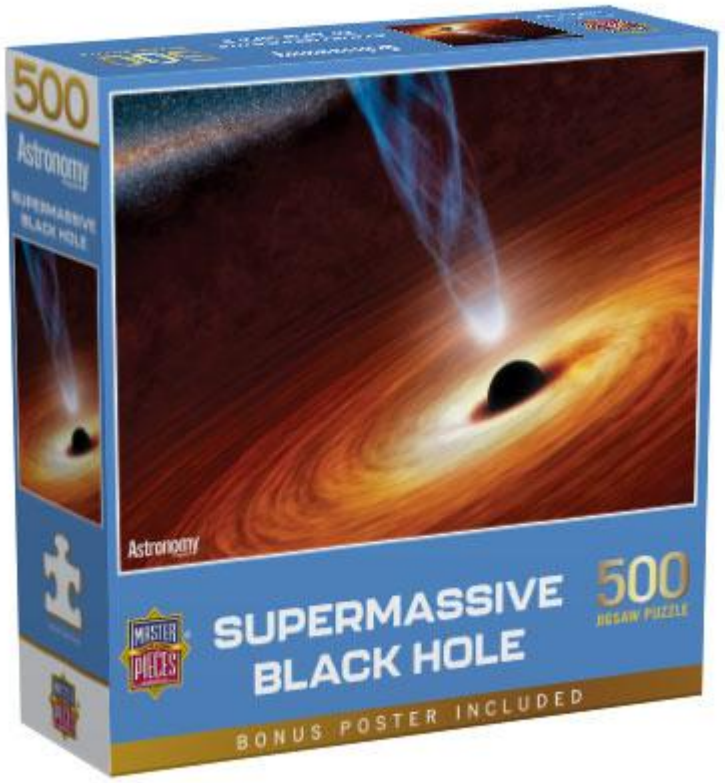
- Diffeomorphism invariance implies that all conserved charges and thermodynamic quantities in GR live on codim-2 surfaces
- "Surface character", evident from BH entropy
- Thermodynamic pressure and volume should also be associated to codim-2 surfaces, not codim-1 surfaces!
- "Size" of BH given by area of the box in which you put it
- Very different from ordinary thermodynamic systems, e.g. gas in a box
- Holographic nature of gravity

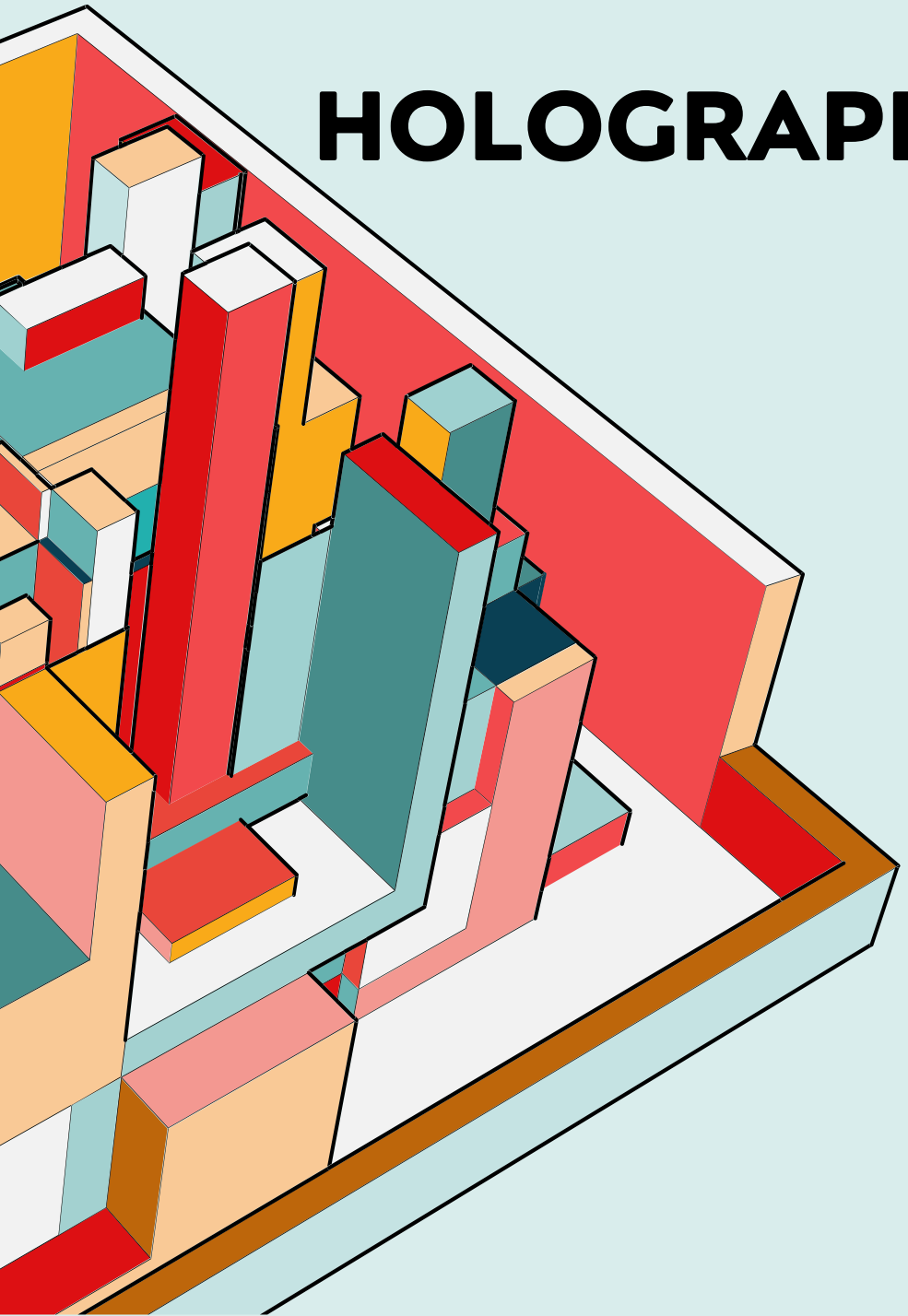




YORK'S QUASI-LOCAL PROPOSAL

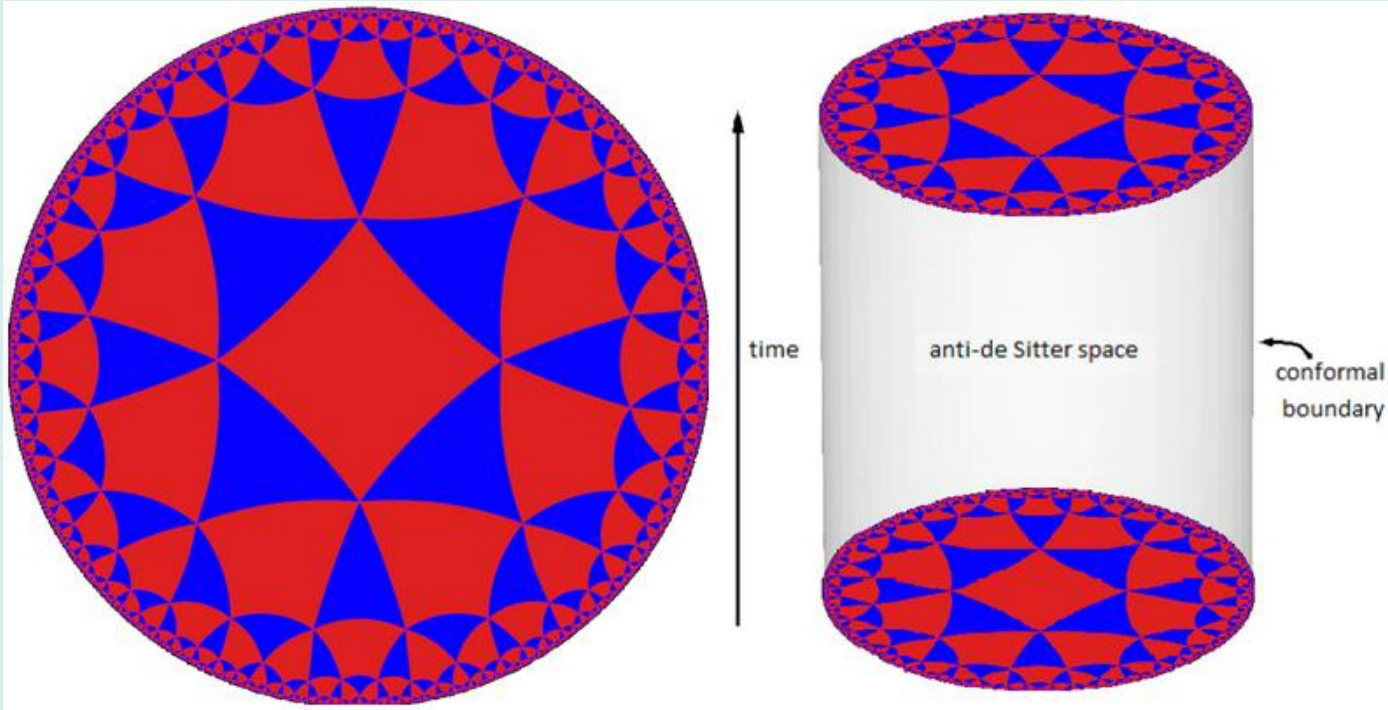
- Black hole in a box
- Cross section of timelike boundary
- Volume is AREA of cross section A
- Pressure is surface pressure s
- Tolman temperature T
- Quasi-local energy E
- First law $dE = TdS - sdA$



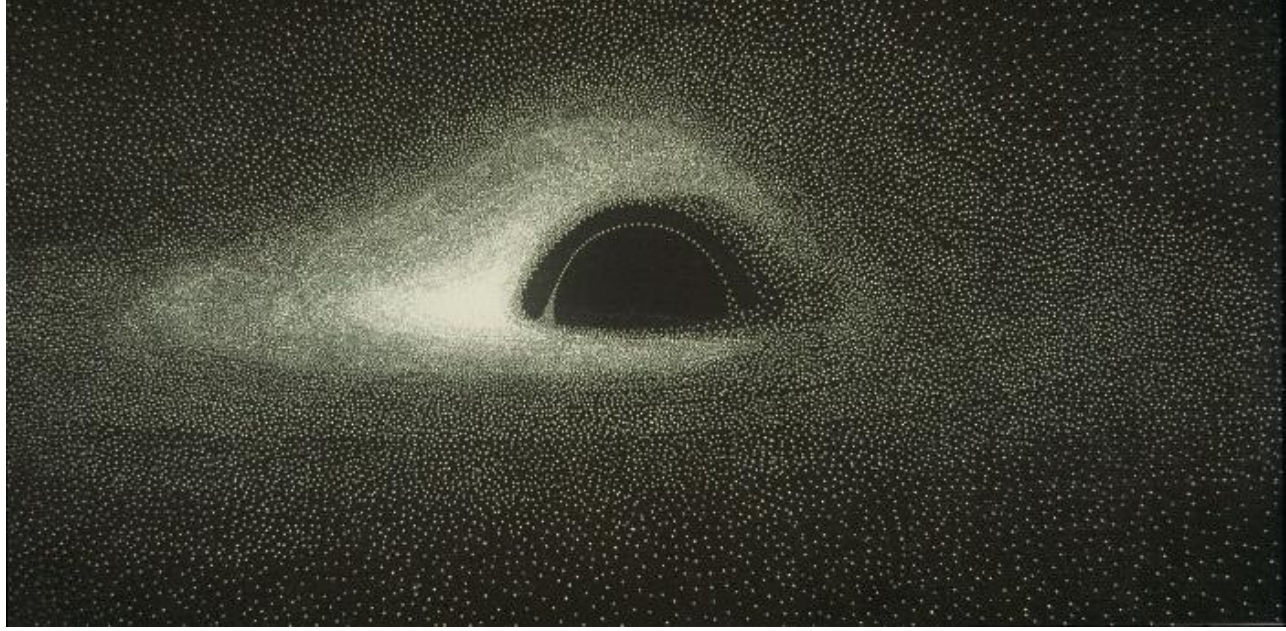


HOLOGRAPHIC PRINCIPLE

- ‘t Hooft
- Information of region encoded on boundary
- AdS/CFT correspondence
- Interpret York’s first law as $dE = TdS - PdV$



SMARR AND EULER RELATIONS



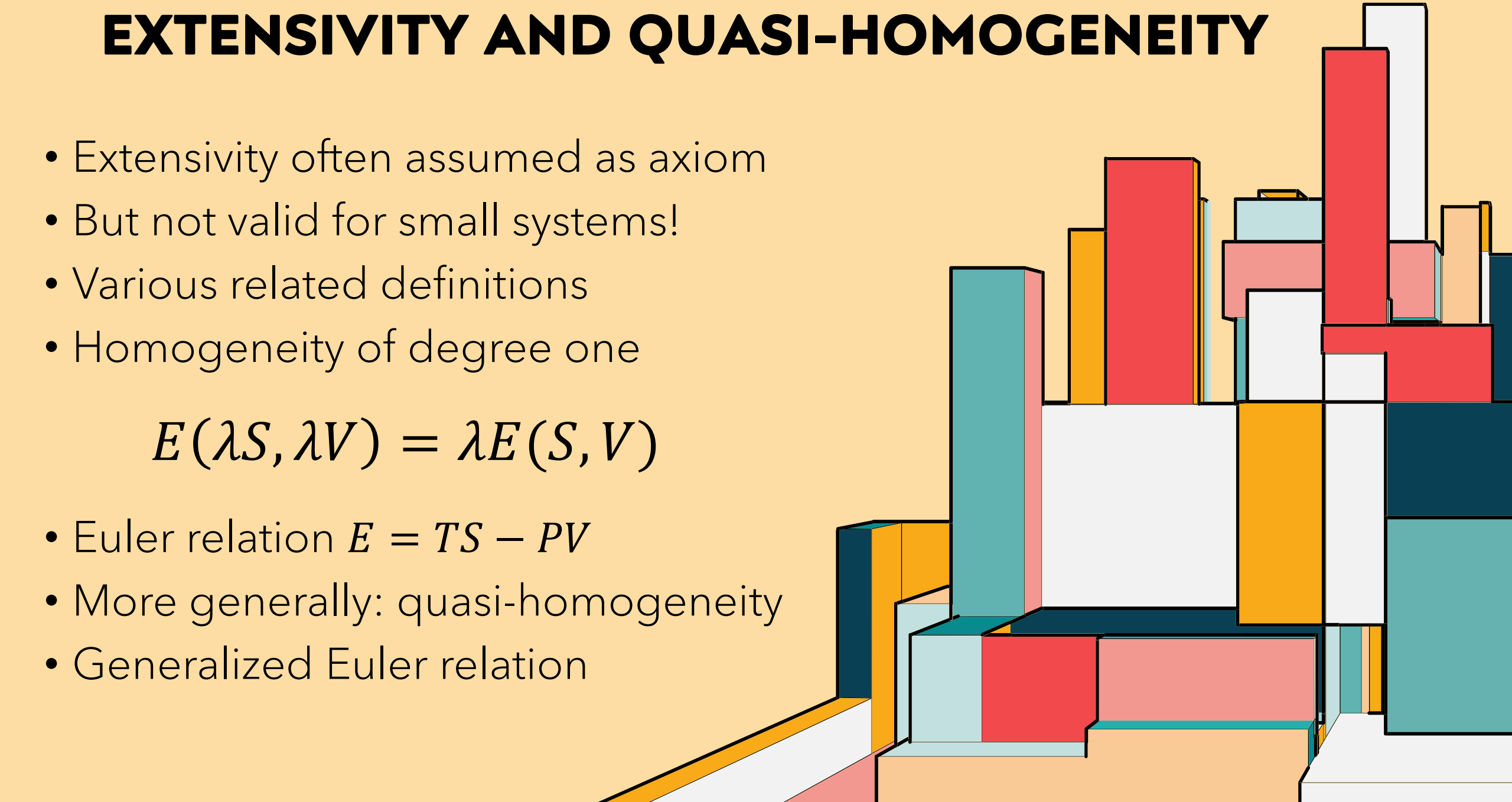
- Schwarzschild: $(d-3)E = (d-2)(TS - PV)$
- AdS-Schwarzschild: $(d-3)E = (d-2)(TS - PV) - \frac{\Lambda \bar{V}}{4\pi GN}$
- How to interpret this extra term?
- Extended BHT
- AdS/CFT: dual to $E = TS + \mu C$
- Generalized Euler relation: correction due to geometric scale
- Our proposal: failure of (quasi-)homogeneity due to small system size

EXTENSIVITY AND QUASI-HOMOGENEITY

- Extensivity often assumed as axiom
- But not valid for small systems!
- Various related definitions
- Homogeneity of degree one

$$E(\lambda S, \lambda V) = \lambda E(S, V)$$

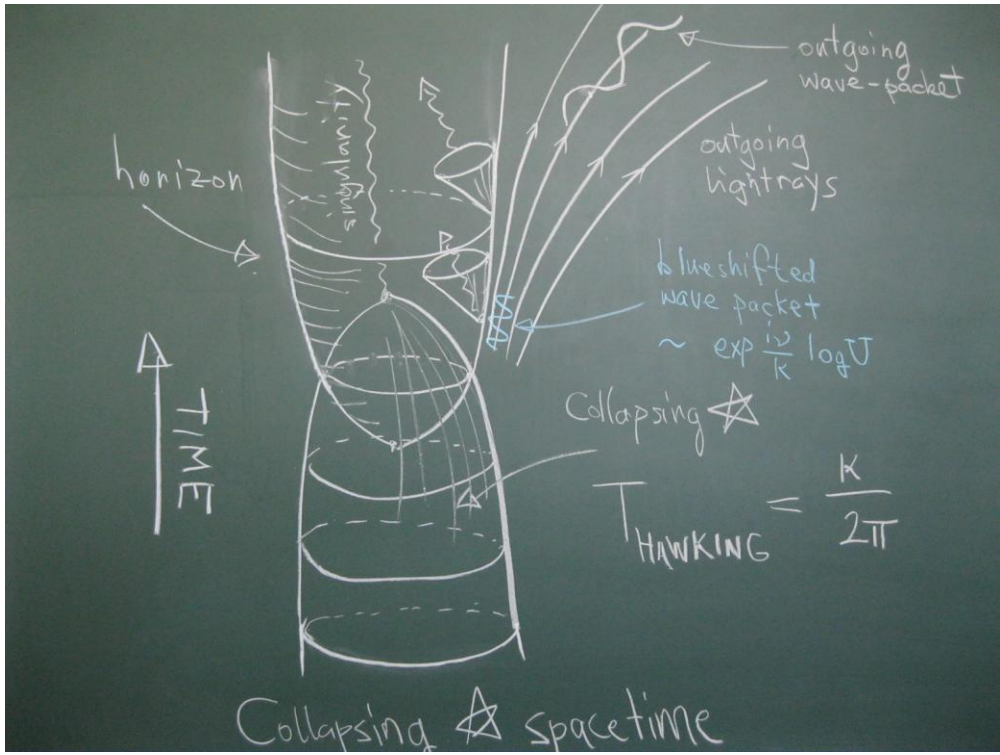
- Euler relation $E = TS - PV$
- More generally: quasi-homogeneity
- Generalized Euler relation



EXTENSIVITY OF ADS-SCHWARZSCHILD BLACK HOLES

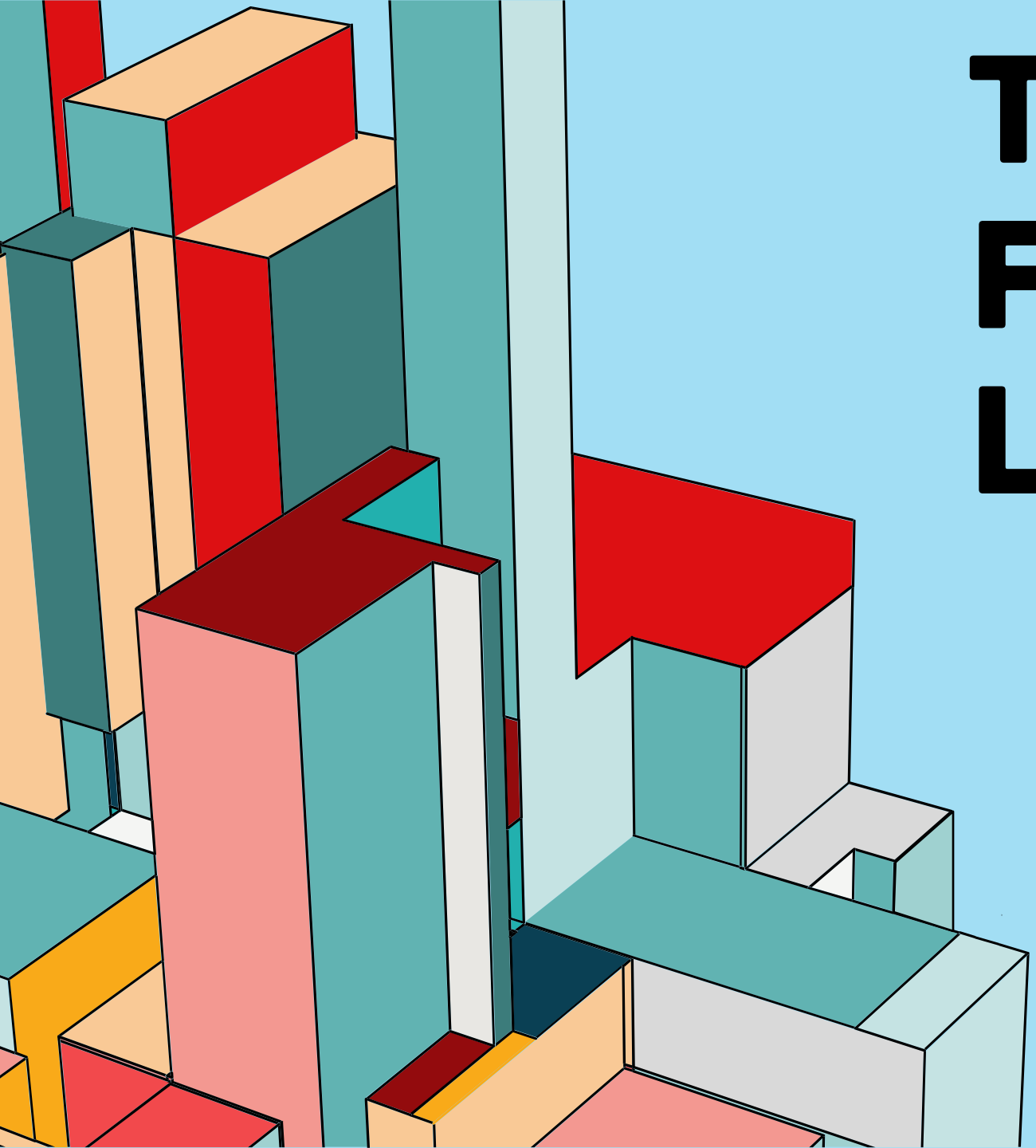
- Schwarzschild: non-extensive
- $(d-3)E = (d-2)(TS - PV)$
- Failure of Euler relation
- Quasi-homogeneity
- AdS-Schwarzschild: extensive in CFT limit
- More general extensive limit
- Formula for extensive energy:

$$E_{\text{ext}}(S, V) = \frac{(d-2)V}{8\pi GL}\left(1 - \sqrt{1 - \left(\frac{4GS}{V}\right)^{\frac{d-1}{d-2}}}\right)$$



DISCUSSION

- Quasi-local thermodynamics combined with holographic principle gives meaningful pressure and volume
- Asymptotically flat BHs are non-extensive even in large-system limit
- Indicates non-locality of dual theory
- Asymptotically AdS BHs have extensive large-system limit more general than CFT
- Closed form expression for extensive energy
- Extensions: charged and rotating BHs
- Relation to other approaches



THANKS FOR LISTENING